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Report on Travel in Europe, Summer 1952

by Fritz John

Purpose of the trip was to gather information on work done at various Institutes in the fields of Numerical Analysis, Partial Differential Equations, and Applied Mathematics with special emphasis on Hydrodynamics. The following report describes the visits made in chronological order.

June 26-July Crossing on Ryndam (Holland America Line) from New York to Rotterdam. Arrived in Amsterdam July 5.

July 5-July 9 in Holland

Main purpose of stay in Holland was a visit to the Mathematisch Centrum of the University of Amsterdam. In addition the Mathematical Institute of the Rijks University in Utrecht and the National Aeronautical Research Institute (NLL) in Amsterdam were visited. The first evening was spent with Schouten, who is secretary of the Board for the Center and acted as director in the absence of van der Corput. One of the points brought out in the philosophy underlying the management of the Center is the importance of having a good all around representation of all branches of Mathematics, including the purest ones, if the Center is to be successful in its computing services. Schouten has done a considerable

amount of work on Partial Differential Equations in a direction that has been rather neglected at the N.Y.U. Institute, namely the study of general systems of equations. The results are contained in the book on "Pfaff's problem and its generalizations" by Schouten and Kulk. I gather that there is a considerable gap between this theory, which is based on the work of Riquier, Cartan and Thomas, and aspires to treat the most general systems in the field of analytic functions, and the more modern theory emphasized at N.Y.U., which is more closely associated with functional analysis and physical applications, and works with real functions. The latter theory is mostly restricted to single equations in one unknown or to determined systems of first order equations, in which the algebraic difficulties encountered are at a minimum. However, the lack of a satisfactory theory for more general systems of equations is felt in many applied problems, e.g. in elasticity and meteorology. The general theory of the type of Schouten's work can doubtlessly be helpful there for orientation in the small, study of characteristic manifolds consistency, etc., but with its restriction to analytic solutions it is still insufficient to give an insight into the behavior of solutions in the large and into the all important question of choice of "appropriate" boundary conditions.

The center consists of the four divisions of Theory, statistics (under van Danzig). Applied Mathematics, and Computing (under van Wijngarden). The Applied Mathematics, once led by Timman, was at the time without a head, van der Waerden not being available for this position. The responsibility for keeping the Center useful from the point of application appears to fall largely on van Wijngarden, who makes an excellent impression. Among the topics discussed with van Wijngarden was a method of his simplifying period analysis on digital computing machines. The main idea consists of using a finite dimensional function space spanned by periodic step functions of amplitude 1, instead of using the ordinary trigonometric functions, and of then approximating them by stepfunctions for numerical purposes. -The Center has carried out a number of boundary value problems for Partial Differential Equations, notably problems involving boundary conditions for the potential equation on an unknown free surface, which arise in connection with important practical questions concerning underground distribution of salt and fresh water. The absence of an electronic machine restricts the Partial Differential Equation problems to those that can be solved by methods like conformal mapping, reduction to 1-dimensional integral equations, or relaxation.

Other Dutch mathematicians, with whom I had longer discussions were Koksma of the Mathematical Center, Popken of the Rijks University of Utrecht, Burgers of the Technical High school at Delft, and Timman of the N.L.L. Among the subjects closer to my line of interest were certain 4th order partial differential equations encountered by Burgers in connection with problems in turbulence, and boundary layer problems, which Timman was in the process of solving numerically. Timman considered a 3-dimensional flow problem, leading to a system of 2 first order quasi-linear equations, with an unusual boundary condition, which consisted in prescribing the behavior of the solution at a single point, the stagnation point of the flow. -Timman had just received an appointment as professor in Delft. He had previously been at the Center, where still some computational work was carried out for him in connection with flutter problems in compressible flow by expansions in Mathieu functions. With his ability and interest in Applied Mathematics he might be a person well worth inviting for a visit to the U.S.A.

July 10-17 in Göttingen

Arrived in afternoon of July 10, just in time to attend lecture by Wielandt of Tübingen on bounds for eigenvalues, and to meet most of the local mathematicians at this occasion. Among other visitors present during part of my stay are Wahler from England and my colleagues Courant and Friedrichs.

The people I saw most of were Rellich, Siegel, Schneider, Stellmacher, Lyra, Stöhr and Heinz. The first named being the person best informed on the subjects I am interested in. Stöhr's interests seemed to have been rather in pure mathematics, but he had written some papers on the fundamental solution of partial difference equations, which I happened to have reviewed for the Math. Reviews, and he seemed to have acquired a taste for problems arising in numerical analysis. He was contemplating a visit to the U.S. and consulted me at length about how to apply for a position in this country. I advised him to get in touch with the Bureau of Standards' Applied Mathematics Laboratory. I spent more time with Stellmacher, who appeared to be quite an expert on the literature on hyperbolic partial differential equations, and had written some notes on the subject. He had done a considerable amount of work in an attempt to prove Hadamard's conjecture on the occurrence of diffusion of waves associated with 2nd order hyperbolic equations. I discussed with him the work of my colleague Douglis on the same subject and its relation to the earlier attempts of Mathiesson. (In the meantime the problem appears to have been settled completely by Douglis.) Stellmacher has an interesting approach to special systems of equations using the notions of conformal geometry. He also has extended some of the results of H. M. Martin and Diaz obtained by their new approach to

Riemann's method of integration. He was very anxious for a chance to work in this country. Contributing factors were the lack of security in his job in Göttingen, and the fact that he is rather isolated with his work in Germany, which appears to be much closer to some of the activities going on here.

Heinz told me about a remarkable inequality he derived for the Gauss curvature of a minimal surface defined over a circle.

Neither the town of Göttingen nor the mathematical Institute seem to have changed much in the 19 years that I had not seen them. Only the total loss of the splendid mathematical library of the Institute makes itself felt.

I visited a number of related institutions in Göttingen. The Geophysical Institute on the Hainberg I saw with Courant. Had discussion with Bartels and his assistant Kertz on work going on there. Most of it appears to be concerned with seismic waves and with magnetic observations, which show e.g. the 27 day period of revolution of the sun. Bartels has some doubts about the feasibility of weather prediction by numerical computation, based on the fact that the significant features that influence the weather most are concentrated on surfaces ("fronts"), which are difficult to handle by finite difference methods.

In addition, I talked with Tolmien and Bierman at the Max Plank Institute. Tolmien shows me their experimental setup, various wind tunnels and tanks. Bierman discusses his numerical work, which consists mostly of computation of wave functions and of particle paths in the earth magnetic field. Computations are carried out on their electronic machine, which has a rather limited magnetic drum memory at present, but which is to be superseded soon by one with a 2000 word (32 binary digits each) memory.

I give a talk in the Mathematische Gesellschaft on derivatives of weak solutions of linear elliptic differential equations.

July 18-19 Vacations

July 20

I set out for Italy via Frankfurt and Heidelberg. Arrive in afternoon of 20th in Frankfurt and spend rest of day and following morning in straightening things out for my trip. Get identification card at Headquarters, IG Farben, and Turkish visa from consul for Mechanics Congress. Then to Heidelberg, where I talk on elliptic differential equations. Meet there with F. K. Schmidt and Courant to discuss projected series of monographs on partial differential equations, to be published by Springer as part of the "Ergebnisse der Mathematik". Back to Frankfurt in evening, to catch MATS night plane to Rome.

July 21-24 in Rome

Visit Naval Attache, Captain Marshall, where I have more copies of my travel orders made. Otherwise I spend most of my time at the Istituto Nazionale per le Applicazioni del Calculo of the Consiglio Nazionale delle Riccerche. See Fichera (known to me from the Institute for Numerical Analysis in Los Angeles), Ghizetti, Gross and some of the younger people. Picone was not in town. In the meantime the excellent report of J. Weyl on the I.N.A.C. has appeared, describing the activities of that Institute completely. I get the impression, going through the list of problems worked there, that Gross does a good deal of the actual analysis of the problems submitted for computation by outside agencies. Fichera is only connected on a part-time basis with the Institute, and Ghizetti is quite versatile in Analysis, working on mapping problems, Hermite polynomials, ordinary differential equations and related subjects.

Computations are carried out on desk machines. For solutions of partial differential equations they have to rely on relaxation methods or on Picone's method of moments.

The computing Center sponsored by Unesco was in a dormant state, apparently for lack of funds.

Have lengthy discussions with Fichera about a method of his for constructing complete sets of solutions of elliptic equations. Also with Ghizetti, who demonstrates a neat method

for solving the Dirichlet problem for the interior of an ellipse, by mapping the slit ellipse on a circular ring and using Fourier series in the ring, taking into account the symmetry conditions arising on the inner circle. I also got interested in a theorem of Ghizetti concerning equivalence of 2 function spaces connected with expansions in Hermite polynomials, which should be related to a similar question arising for solutions of a wave equation subject to a Sommerfeld radiation condition.

Fichera would like to have somebody from Courant's group, e.g. P. Lax, to spend a year in Italy on a Fulbright Fellowship half of the time in Trieste.

The I.N.A.C. was in the process of closing up for a month's summer vacation. Every day it became more difficult to see people in Italy. I had hoped to have an opportunity to see Mrs. Cinquini-Cibrario, who is one of the pioneers in the theory of hyperbolic equations in 2 independent variables. I did not have her correct address, and she apparently had gone on vacation, before any of my letters reached her.

July 25-27 Naples

Travel on Friday morning, July 25, from Rome to Naples by train, mostly to see Miranda and his group at the University of Naples. I find that the aspects of the theory

of partial differential equations of most interest to them are those connected with variational problems. They consider rather special systems of equations arising in this way, with emphasis on assuming as little as possible in the way of regularity on the coefficients. Results for such systems are obtained by approximating the equations by equations with more regular coefficients. This involves working with fine a priori estimates for the solutions and their first derivatives in terms of moduli of continuity and Hölder conditions for the coefficients. The work follows the pattern of the results of Morrey at the University of California. -Miranda was surrounded by a number of younger people, Stampachia, Greco, Ciliberto, who made excellent impression in discussion. Unfortunately, I was not given much of an opportunity to discuss their work with them. -Caccipoli, who did important work on elliptic differential equations, was not in town.

July 28

Left for Pisa via Rome, by train. I saw Conti and Baiada there. I had been interested in Conti's work on equations of mixed type, as they occur in gas dynamics. Conti himself is not familiar with the physical applications. He and Baiada told about some more recent work of theirs on special systems of first order equations, with the emphasis again on a minimum of regularity assumptions. This seems to

be the general fashion among the younger people in Italy working on differential equations, with the exception of Fichera. There is little interest in methods applicable to more general types of equations with more regular behavior. Indeed, I found that people look down upon the methods and results of the older generation represented e.g. by Pantappie, who still work with such outmoded tools as analytic functions. According to Baiada the ideal theory of partial differential equations would be patterned after that of the ordinary differential equations, with existence theorems established beyond the point, where uniqueness holds. This might be brought about by considering suitable classes of "weak" solutions.

Return to Rome, July 30, take MATS plane to Frankfurt the same day, arriving there at 11:00 p.m.

July 31-Aug. 1st

Institut für praktische Mathematik in Darmstadt.

Here, under direction of Prof. Walther, the emphasis is considerably more on the practical aspects of Mathematics then in Rome or at the Center in Amsterdam. An electronic machine is being built by Mr. Dreyer. In use was their Differential Analyzer. It made the impression of a rather bulky machine, but works supposedly with great precision. It has only 4 integrators, in addition to the multiplying unit, and 3 output-input tables. There are improvements over the purely mechanical type of machine, I recalled from

Aberdeen, in that use is made of servo-mechanisms and optical following devices. Most interesting for me from the point of view of numerical analysis was a talk with Dr. Under, who is an expert on inversion of matrices, and computation of eigenvalues for matrices with simple or non-simple elementary divisors. They have inverted 10 by 10 matrices at this Institute, but feel that 20 by 20 would be about the maximum size they can handle on their desk calculators. Under is inclined to consider the Gauss scheme for inversion as more efficient than iteration schemes. He was interested in Lanczos' method for determination of eigenvalues. He was also preparing a new type of table for confluent hypergeometric functions. The exact values would be given for complex integers only; the values at other points would be obtained by interpolation formulae based on derived polynomials, making use of the differential equations satisfied by the functions.

Walther is very interested in visual aids for teaching mathematics, and is quite inventive in devising models and gadgets for that purpose. The institute supports itself largely through its work for industry. They are seriously handicapped by their lack of computing machines. Even I.B.M. machines would bring an improvement over their staff of 6 human computers.

On August 1st I leave for Göttingen, where I meet Rellich. We discuss at great length the work of H. Weyl and K. Müller on capacity of radiation fields. Weyl had given a talk in Göttingen on that subject the week before. I was specially interested, because I had encountered similar problems in dealing with water waves in the presence of obstacles.

August 2-16 Vacation

August 17 - Start trip to International Congress in Istanbul.

Arrive in Frankfurt in afternoon of 17th. First MATS plane I can get on leaves on August 19 at 2:30 a.m. for Athens. MATS plane takes off from there to Tripoli taking along all my baggage through an oversight, including the slides for my talk at the Congress. No MATS transportation in sight from Athens to Istanbul or Ancara for next 3 days at least. As meeting is to start next day, taking accommodations on commercial French Airline plane to Istanbul, where I arrive at 9 p.m. of the 19th. Have difficulty in finding accommodations for first night, as I only have room reservation from next day onwards. The Turkish Committee for the Congress is extremely helpful in solving this and any other difficulties arising.

The sessions of the Congress are distributed over 5 sections, which meet simultaneously. Except for general sessions I spend all my time in the section dealing with

Hydrodynamics. Other sections are covered in the report of J. Keller. Of particular interest to me was a paper by Féseau, giving an explicit expression for linearized flows over uneven bottom surfaces of certain special shapes. The method of solution employs reduction to difference equations by Fourier transformation, similar to those used by Peters in other problems. A paper by Y. Yamamoto, though presented by proxy in a rather incomprehensible fashion, showed some overlapping with work contained in an N.Y.U. thesis by Finkelstein.

I met and talked with a large number of people from here and abroad. With Kravtchenko of Grenoble, who acquainted me with the work of Gerber, establishing uniqueness and existence of waves over periodic bottoms. (Gerber also has given a proof of the analyticity of the free surface of a steady wave independently of H. Lewy.) With Peres of Paris, who gave me introductions to two French hydrodynamical laboratories. With Weinblum of Hamburg, with whom I discuss possible applications of my work on waves in the presence of obstacles. With Hogner of Stockholm on wave resistance. Other conversations with Lighthill of Manchester, Willharz of Freiburg, Milne-Thompson of the Royal Naval College.

Give paper on construction of plane flows with a free surface and gravity present.

August 27

Leave Istanbul for Athens with one of 2 MATS flights arranged by Militar Attachees for the American participants of the Congress. Held up in Athens till 29th for lack of further transportation to Frankfurt. Arrive in Frankfurt the evening of the 29th.

August 29-Sept. 1st Vacation

September 2nd leave for Skandinavia

September 3rd in Kopenhagen. See Fenchel. Discuss convex geometry, determination of convex surfaces from given level lines. Arrive in the evening of the same day in Lund (Sweden).

September 4-7 in Lund

Lund is a typical university town with the physical and mathematical Institues occupying a large modern building. Most of the time I saw Gårding, Marcel Riesz and Pleijel all of whom I had met previously in the U.S. All three of them are concerned with various aspects of the theory of partial differential equations, which also are of interest to the N.Y.U. Institute. Gårding has made important contributions to the theory of the general linear elliptic equations, discovering some inequalities, which permit to extend the techniques of K. O. Friedrichs to the most general cases, and to solve the Dirichlet problem. M. Riesz was working on a systematic discussion of characteristic surfaces and

corresponding initial value problems for the wave equation, some instances of which are also treated in a recent N.Y.U. thesis by Gardner. He has succeeded in giving an abstract general definition of characteristic surfaces, by completion of point sets consisting of characteristic generators. One of his results consists in giving an elegant expression for solutions of the wave equation having the same type of singularity along an arbitrary characteristic surface, as that possessed by the fundamental solution along a characteristic cone. -Pleijel is mostly concerned with distribution of eigenvalues for higher order equations. -I acquaint them with some recent work on existence of derivatives of weak solutions of elliptic equations and on convergence of difference methods for numerical determination of solutions of partial differential equations. -Riesz expresses some interest in having an opportunity to teach a course in the U.S. on differential geometry or related subjects.

September 8-12 Paris

I pay a visit to the Laboratoire Central d'Hydraulique de France, which is under the direction of Jean Laurent. Am received by P. Gerlier in the absence of the director. A large part of their space was occupied by 3 models of sections of the course of the Seine River between the mouth and Rouen, at different scales. The purpose was to study the bore

arising at the mouth of the river in consequence of the tides and to derive means of making its effect in the harbor of Rouen harmless. Their model apparently reproduces the progress of the front of the bore with extreme accuracy. Metal gratings are used to give the required frictional effects. I had some discussion with them about the theoretical computation of bores. They were unacquainted with Stoker's theory of bore formation in analogy to shockwaves in gases. One reason was that they had been unable to obtain the first number of the Communications on Pure and Applied Mathematics, which contains Stoker's paper and is out of print. They make use instead of an older theory by Favre, and also claim that the bore can be treated as a solitary wave with good approximation, in spite of the obvious lack of symmetry of the waves. The result of their experiments was that the bore in the harbor can be killed off by adding a suitable extra basin. -Another experimental arrangement I found of great interest dealt with the breaking of waves. The relative motion of the fluid particles is made visible by insertion of pieces of paper and photographed from a carriage proceeding with the speed of the wave. It is then observed that breaking sets in at the point where the relative speed at the surface is zero. Other experiments dealt with measurements of tension in ships cables due to a bore. -They publish the Revue Generale de l'Hydraulique, and induce me to promise somewhat hastily the contribution of some article.

The other institute visited by me was The Laboratoire National d'Hydraulique of the Electricite de France at Chatou, where I am conducted by a Mr. Valembois. They were largely concerned with study of transport of materials at the bottom of harbors, due to wave action. They have devised an interesting method for obtaining almost complete reflection of the waves of a certain length through use of side basins of dimensions corresponding to the wave length. If a series of such basins is installed, covering a large enough range of wave lengths, the water will be practically at rest beyond.

The only mathematician I see in Paris is J. Hadamard, who was busy working on a book incorporating recent progress in differential equations.

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John